

For $V = 10 \cos 2500t$, Find the power factor for both circuits.

$$\begin{aligned} I &= \frac{V}{Z} = \frac{V}{1k \parallel j\omega L} \\ &= \frac{10 \angle 0^\circ}{707.11 \angle 45^\circ} \\ &= 14.14 \angle -45^\circ \text{ mA} \end{aligned}$$

$$\begin{aligned} \text{PF angle} &= \theta_V - \theta_i \\ &= 0 - (-45) \\ &= 45^\circ \end{aligned}$$

$$\text{PF} = \cos 45^\circ = 0.707 \text{ Lagging}$$

$$\begin{aligned} I &= \frac{V}{Z} = \frac{V}{1k \parallel \frac{1}{j\omega C}} \\ &= \frac{10 \angle 0^\circ}{371.39 \angle -68.20^\circ} \\ &= 26.93 \angle 68.20^\circ \text{ mA} \end{aligned}$$

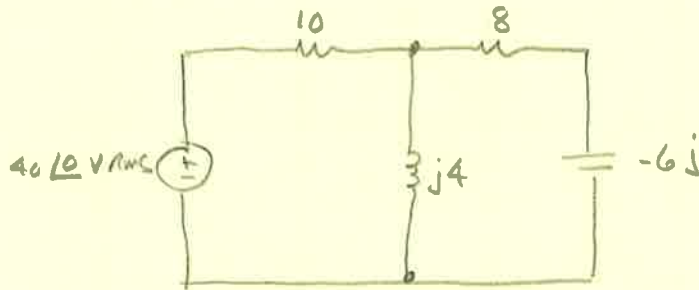
$$\begin{aligned} \text{PF angle} &= \theta_V - \theta_i \\ &= 0 - 68.20^\circ \\ &= -68.20^\circ \end{aligned}$$

$$\text{PF} = \cos(-68.20) = 0.371 \text{ leading}$$

Note: $\text{PF} = \cos(\angle Z_{\text{eq}})$

Chap 10 Lecture Power Factor

Find PF as seen by the source



$$Z = 10 + j4 \parallel (8 - 6j)$$

$$= 10 + \frac{j4(8 - 6j)}{j4 + 8 - 6j} = 10 + \frac{32j + 24}{8 - 2j} = 10 + \frac{40 \angle 53.13^\circ}{8.2462 \angle -14.0362^\circ}$$

$$Z = 10 + 4.8507 \angle 67.1663^\circ = 10 + 1.8824 + 4.4706j$$

$$= 11.8824 + 4.4706j = 12.6955 \angle 20.6182^\circ$$

$$PF = \cos \angle Z = \cos(20.6182^\circ) = \boxed{.936 \text{ (lagging)}} \quad \angle \theta Z \text{ is positive}$$

$$P_{AVG \text{ source}} = V_{RMS} I_{RMS} \cos(\theta_v - \theta_i)$$

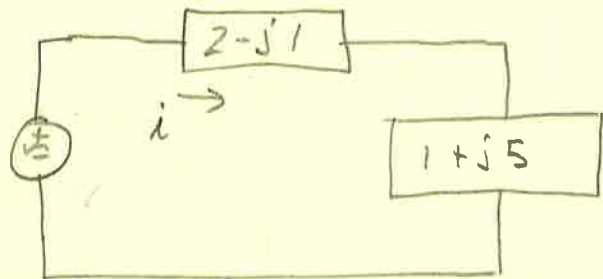
$$i = \frac{V}{Z} = \frac{40}{12.6955} \angle -20.6182^\circ$$

$$= 40(3.1507) \cos(0 - (-20.6182^\circ))$$

$$= \boxed{118W}$$

- 1) FIND P_{avg} for both z_1
 2) " P_{app} supplied by source
 3) PF

$$60 \angle 0^\circ \text{ RMS}$$



$$1) \hat{i} = \frac{V}{Z} = \frac{60 \angle 0^\circ}{2-j1+1+j5} = \frac{60 \angle 0^\circ}{3+j4} = 12 \angle -53.1301^\circ \text{ A}_{RMS}$$

$$V_{2-j1} = I Z = (12 \angle -53.1301^\circ)(2-j1) = 26.8328 \angle -79.6952^\circ \text{ V}_{RMS}$$

$$V_{1+j5} = I Z = 61.1082 \angle 25.56^\circ \text{ V}_{RMS}$$

$$P_{avg(2-j1)} = V_{RMS} I_{RMS} \cos(\theta_v - \theta_i) = \boxed{288 \text{ W}}$$

$$P_{avg(1+j5)} = \boxed{144 \text{ W}}$$

$$\text{also, } P_{avg_2} = I^2 |z_{real}| = (12)^2 / 2 = \underline{288}$$

$$P_{avg_1} = I^2 |z_{real}| = (12)^2 (1) = \underline{144}$$

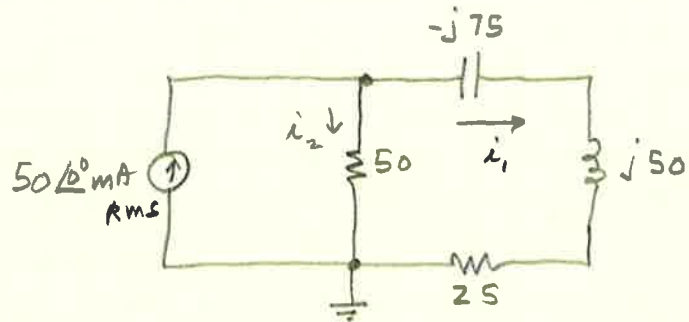
$$2) P_{app} = V_{RMS} I_{RMS} = 60(12) = \underline{720 \text{ VA}}$$

$$3) PF = \cos(\theta_v - \theta_i) = \cos(0 - (-53.1301^\circ)) = \boxed{0.6 \text{ lagging}}$$

$$\text{ALSO, } PF = \angle \text{ of } Z = \frac{V}{I}$$

$$\angle Z = (2-j1) + (1+j5) = 3+j4 = 5 \angle 53.1301^\circ$$

$$\text{also, } PF = \frac{P_{avg}}{P_{app}} = \frac{288+144}{720} = \underline{0.6}$$



a) Find P_{avg} + P_{react} for the source

$$\begin{aligned} Z_{eq} &= 50 \parallel (-j75 + j50 + 25) \\ &= \frac{50(25 - 25j)}{50 + 25 - 25j} = \frac{50(35.36 \angle -45^\circ)}{79.06 \angle -18.43^\circ} \\ &= 22.36 \angle -26.57^\circ \Omega \\ &= 20 - j10 \Omega \end{aligned}$$

$$\begin{aligned} V_{source} &= I_s Z_{eq} = 50 \angle 0^\circ (22.36 \angle -26.57^\circ) \\ &= 1.12 \angle -26.52^\circ \\ &= 1 - .5j \end{aligned}$$

$$\begin{aligned} P_{avg} &= V_{rms} I_{rms} \cos(\theta_v - \theta_i) \\ &= 1.12(50 \text{ mA}) \cos(-26.52^\circ - 0) \\ \boxed{P_{avg} = -50.0 \text{ mW}} \end{aligned}$$



$$\text{PF angle} = \angle Z = -26.57^\circ$$

$$\begin{aligned} P_{react} &= V_{rms} I_{rms} \sin(\theta_v - \theta_i) \\ \boxed{P_{react} = +25 \text{ mVAR}} \end{aligned}$$

b) current source is delivering $\boxed{50 \text{ mW}}$ of average power

c) " " " absorbing 25 mVA " reactive "

d) Find complex power for each element

$$i_1 = \frac{50 \angle 0^\circ \text{ mA} (50)}{50 + 25 + j50 - j75} = \frac{25 \angle 0^\circ}{75 - 25j} = \frac{2.5 \angle 0^\circ}{79.01 \angle -18.43^\circ} = 31.6 \text{ mA} \angle 18.43^\circ = 30 + j10$$

$$i_2 = i_s - i_1 = 50 \angle 0^\circ - 31.6 \angle 18.43^\circ \text{ mA} = 20 - j10 = 22.36 \angle -26.57^\circ \text{ mA}$$

$$P_{50\Omega} = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = (1.12)(22.36) \text{ mW} = \boxed{25.0 \text{ mW} = P_{50\Omega}}$$

$$P_{25} = I^2 R = (31.6)^2 25 = \boxed{25 \text{ mW} = P_{25\Omega}}$$

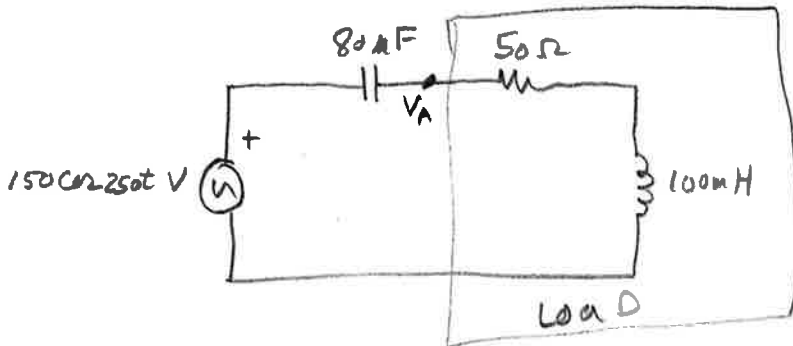
$$\begin{aligned} P_{cap} &= V_{rms} I_{rms} = I_{rms}^2 Z \cos(\theta_v - \theta_i) \\ &= (31.6)^2 (-75) = \boxed{-75 \text{ mVAR} = P_{cap}} \end{aligned}$$

$$P_{ind} = I_{rms}^2 Z_{IND} = (31.6)^2 50 = \boxed{50 \text{ mVAR} = P_{ind}}$$

e) P_{avg} delivered = -50 mW ; P_{avg} absorbed = 50 mW ✓

f) P_{react} " = 25 mVAR ; P_{react} " = $-75 + 50 = -25 \text{ mVA}$ ✓

Find P_{avg}
 Reactive
 PAPP
 absorbed by the load.



1) Put circuit in phasor form:

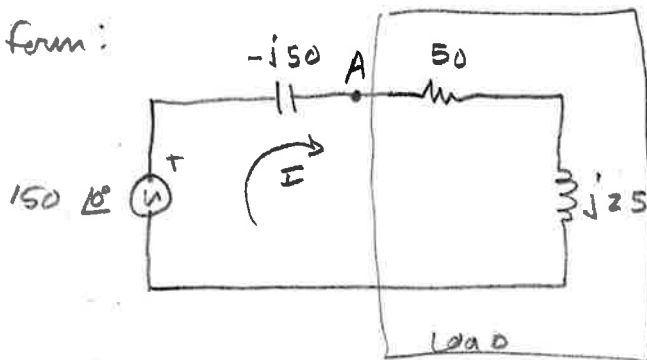
$$Z_C = \frac{1}{j\omega C} = -j50$$

$$Z_L = j\omega L = j25$$

$$I = \frac{150 \angle 0}{-j50 + 50 + j25} = \frac{150 \angle 0}{50 - j25}$$

$$= \frac{150 \angle 0}{55.90 \angle -26.57^\circ} = 2.683 \angle 26.57^\circ \text{ A}$$

$$= 2.4 + j1.20 \text{ A}$$



$$V_A = I(50 + j25) = (2.683 \angle 26.57^\circ)(55.90 \angle 26.57^\circ)$$

$$= 149.98 \angle 53.14^\circ \text{ V}$$

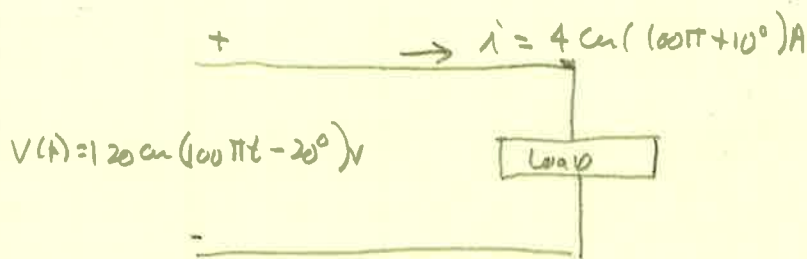
$$\text{Then } P_{avg} = \frac{V_m I_m}{2} \cos(\theta_V - \theta_I) = \boxed{180 \text{ W}}$$

$$P_{rea} = Q = \frac{V_m I_m}{2} \sin(\theta_V - \theta_I) = \boxed{90 \text{ VARs}}$$

$$P_{APP} = P + jQ = 180 + j90 = \boxed{201.2 \angle 26.57^\circ \text{ VA}}$$

$$\text{also, } P_{avg} = \frac{1}{2} I^2 R = \frac{1}{2} (2.683)^2 (50) = 180 \text{ W} \quad \checkmark$$

$$P_{rea} = Q = \frac{1}{2} I^2 Z_L = \frac{1}{2} (2.683)^2 (25) = 90 \text{ W} \quad \checkmark$$



Find the Apparent Power + PF of the load

$$P_{\text{App}} = V_{\text{eff}} I_{\text{eff}} = \frac{1}{2} V_m I_m = \frac{1}{2} (120)(4) = \underline{240 \text{ VA}}$$

$$P_{\text{ave}} = \frac{1}{2} V I \cos(\theta_v - \theta_i) = \frac{1}{2} (120)(4) (\cos(-20 - 10)) = 208 \text{ W}$$

$$\text{PF} = \frac{P_{\text{ave}}}{P_{\text{App}}} = \underline{0.866} \quad (\text{Leading, i.e., the current leads the voltage})$$

$$\text{also, } Z_{\text{load}} = \frac{V}{I} = \frac{120 \angle -20}{4 \angle 10} = 30 \angle -30^\circ = 26.0 - 15j = Z$$

$$\text{PF} = \cos(-30^\circ) = \underline{0.866}$$

Since current leads voltage, PF is leading

$$-15 = -\frac{1}{\omega C} \Rightarrow C = \frac{1}{15\omega}$$

$$C = \frac{1}{15(100\pi)} = 212 \mu\text{F}$$

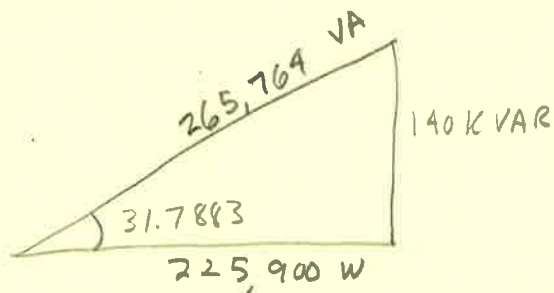
$$R = \underline{26.0 \Omega}$$

chap 10 lecture PF correction

$$V = 110 V_{RMS}, 60 \text{ Hz}$$

$$\text{load} = 140 \text{ KVAR} @ 0.85 \text{ PF lagging}$$

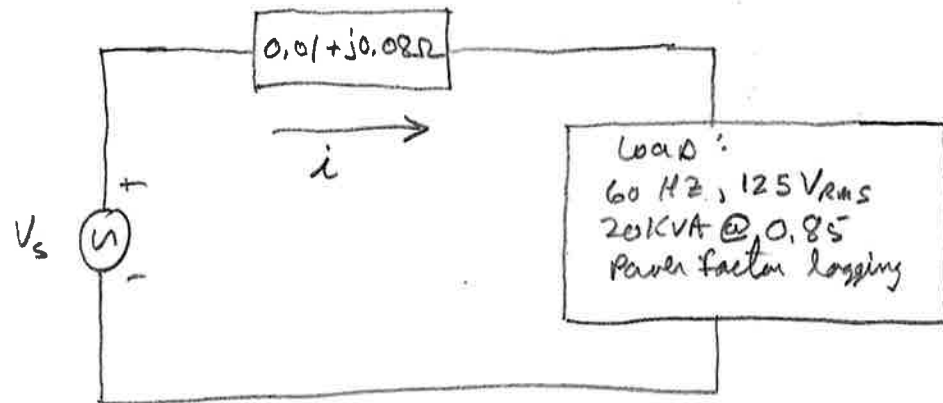
what value of parallel capacitance is needed to correct to a unity PF



The capacitor must provide 140 KVAR of reactive power in order for unity PF

$$Q_c = \frac{V_{RMS}^2}{Z_c} = \frac{V_{RMS}^2}{\frac{1}{\omega C}} \quad \text{or} \quad C = \frac{Q_c}{\omega V^2} = \frac{140 \text{ KVAR}}{2\pi(60)(110^2)} = \underline{\underline{30,610 \mu\text{F}}}$$

$$\begin{aligned} \text{also, } C &= \frac{P(\tan \theta_{old} - \tan \theta_{new})}{\omega V_{RMS}^2} \\ &= \frac{(225,900)(\tan 31.7883^\circ - \tan 0^\circ)}{2\pi(60)(110)^2} \\ &= 30,691 \mu\text{F} \end{aligned}$$



a) find V_{source}

$$\theta = \cos^{-1} 0.85 = 31.79^\circ$$

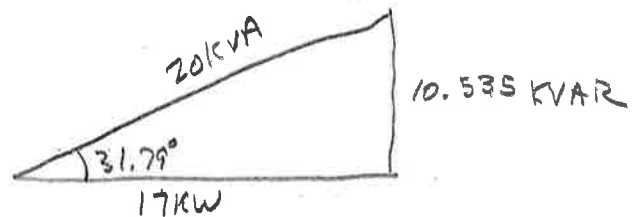
$$S = V_{rms} I_{rms}^*$$

$$17k + j10.535k = 125 \angle (I^*)$$

$$I^* = 136 + j84.29 \text{ A}_{rms}$$

$$I = 136 - j84.29$$

$$V_s = 125 \angle 0 + I(0.01 + j0.08) = \boxed{133.48 \angle 4.31^\circ \text{ V}_{rms}}$$



b) what is the average power loss in the feeder?

$$P_L = I^2 R = (136)^2 (0.01) = \boxed{256 \text{ W}}$$

c) what size caps is need to make $PF = 1$?

$$\text{method 1: } C = \frac{P(\tan \theta_1 - \tan \theta_2)}{\omega V_{rms}^2} = \frac{17k(\tan 31.79 - \tan 0)}{2\pi(60)(125)^2} = \boxed{1789.7 \mu\text{F}}$$

$$\text{also, } \frac{V^2}{X_C} = Q \Rightarrow \frac{(125)^2}{X_C} = -10,535 \text{ VAR}$$

$$\text{But } X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{1/\omega C} = \frac{1}{-10,535 / (2\pi(60)(125)^2)} \Rightarrow C = 1788.7 \mu\text{F}$$

d) after the caps is installed, what is V_{source} ?

after caps is installed, $Q=0$ so $I = 136 \text{ A}_{rms}$

$$V_s = 125 + 136(0.01 + j0.08) = \boxed{126.83 \angle 4.92^\circ \text{ V}_{rms}}$$

e) what is the new P_{avg} loss in the feeder?

$$P_L = I^2 R = (136)^2 (0.01) = \boxed{184.96 \text{ W}}$$